Graphing Calculator and Geometers’ Sketchpad in Teaching and Learning of Mathematics

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ABSTRACT
This study aims to develop a module for teaching and learning of Straight Lines topics using graphing calculator (GC) technology and dynamic geometry software, Geometer’s Sketchpad (GSP) and also to determine the pedagogical usability of the module. The study is carried out in two phases. Phase 1 is the development of the module and phase 2 is a survey research using a questionnaire. A sample of 30 mathematics teachers from three different schools in district of Sabak Bernam, Hulu Selangor and Kuala Selangor are randomly selected to participate in this study. The instrument used in this study is the pedagogical usability questionnaire. The pedagogical usability criteria are the student control, student activities, objective-oriented, application, value-added, motivation, knowledge value, flexibility and response. The data are analysed using descriptive statistics such as mean, standard deviation, percentage, and frequency. The findings showed that the module developed has a high pedagogical usability. Findings from this study provide evidence of pedagogical impact in incorporating the latest trends in mathematics education, namely, integrating the GC tool and GSP software to maximize the mathematical and pedagogical benefits to students.

1. Introduction

Concept building and skills acquisition as well as the inculcation of good and positive values are the main focus in teaching and learning processes for Malaysian mathematics (Ministry of Education Malaysia, 2011). There are also other elements that have been taken into account and infused into the teaching and learning for mathematics as stated in the Malaysian Mathematics Curriculum Specifications, namely, (i) problem solving, (ii) mathematical communication, (iii) making connection, (iv) reasoning, and (v) the use of technology.

Among those elements, the use of technology in teaching and learning of mathematics has consistently been one of the major emphases in Malaysian Integrated Curriculum for Secondary School Mathematics. Technology explosion has inspired various methodologies for the purpose of effective teaching and learning in mathematics. Teachers are encouraged to use the latest technology to help students understand mathematical concepts in depth, to enable them to explore mathematical ideas (Ministry of Education Malaysia, 2011). This emphasis is congruent with the NCTM’s Technological Principle which states that, “Technology is essential in teaching and learning mathematics, it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 24).

The emphasis on integrating technology in the teaching and learning of mathematics is parallel with the aim of the Malaysian mathematics curriculum: to develop individuals that are able to face challenges in everyday life that arise due to the advancement of science and technology (Ministry of Education Malaysia, 2011). However, technology does not replace the need for all students to learn and master the basic mathematical skills. Without the use of technologies such as the calculators or other electronic tools, students should be able to add, subtract, multiply and divide efficiently. The mathematical curriculum therefore requires the use of technology to focus on the acquisition of mathematical concepts and knowledge rather than merely doing calculation.

There are many kinds of technology that are considered relevant to school mathematics these days. These range from very powerful computer software such as Mathematica, Maple, and MathLab to much powerless

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technologies such as the use of calculators and paper and pencil. The use of dynamic geometry software, namely, the Geometers’ Sketchpad (GSP) and the use of hand-held technologies, namely, the graphic calculators (GC) has been explicitly suggested in the curriculum specifications for secondary school mathematics (Ministry of Education Malaysia, 2011). In the case of GC, the Ministry of Education has started distributing the GC to several hundred secondary schools throughout the country since 2002. However, the usage of graphic calculators in Malaysian school is still in the early stage and there are not many schools which have explored the use of the technology (Nor’ain Mohd. Tajudin, 2011; Noraini Idris, 2004; Lim & Kor, 2004). Furthermore, Malaysia has not started on compulsory implementation of using graphic calculator in teaching and learning of mathematics. In comparison, countries such as England, Australia, Singapore, Thailand, Japan and United States of America has longed implement the usage of graphic calculator as early as 1998. Since the scientific calculators are already used in the SPM examination level, it would also be timely to think about using graphic calculators in the context of mathematics teaching and learning and thus in Malaysian public examination. This would bring Malaysian secondary mathematics education to be at par with other countries and thus, it is worth to spend a large amount of money acquiring the handheld devices.

Furthermore, recently the Malaysian Ministry of Education (MMOE) made the unprecedented but bold and wise decision of subscribing to the license of this GSP software in 2004. The license was not only meant for the quarter of a million teachers and educators including lecturers of public universities but also for the almost 5 million students who are under the direct authority of the MMOE nationwide (Ministry of Education Malaysia, 2001). In the same vein, referring the NCGC (2008; 2011), research on the use of GSP software in teaching and learning of mathematics in Malaysian secondary schools were limited and not in depth.

Even though graphing calculators were distributed to several selected schools in 2002 and the license of GSP was subscribed in 2004, not much success has been recorded as to their use in mathematics classroom practices. Most probably, the problem is that most teachers have not learnt mathematics using these technology tools before hence they were lack of knowledge of TPCK on how to use them in mathematics teaching. According to a study by Effandi et al. (2007), two factors have been identified as the main factors in the application of technology in the teaching and learning of mathematics. The first factor is the teachers’ perception that the use of technology is not able to help in the teaching and learning of mathematics. This was further worsened by the fact that teachers always claim that they do not have sufficient time to prepare for ICT integrated lessons. According to a research by Haslina et al. (2000), in the present teachers’ professional development courses, there are hands-on activities but this was not supported by relevant modules or manuals for the facilitators and the course participants. The activities conducted in those courses are teacher-centered and in most situations, courses are conducted using softcopy materials supplied by vendors. The approach was rather ineffective in the learning of particular software which normally requires active participation from the participants.

Thus, the question now is to identify what and how to prepare mathematics teachers to teach in the 21st century. Teachers need guidance to be able to do and how do they need to develop this knowledge for teaching mathematics. In other words, to be prepared to teach mathematics, teachers need an in-depth understanding of mathematics (content), teaching and learning (the pedagogy), and technology pedagogical content knowledge (TPCK) – “an overarching conception of their subject matter with respect to technology and what it means to teach with technology” (Niess, 2005, p.510). TPCK for teaching with technology means that as teachers think about particular mathematics concepts, they are concurrently considering how they might teach the important ideas embodied in the mathematical concepts in such a way that the technology places the concept in a form understandable by students.

Thus, this research attempts to provide the current best practices in integrating the use of GC and GSP software in teaching and learning of mathematics, present and describe instructional strategies using these technologies, hence to develop the GC-GSP module in teaching and learning of Straight Lines Topic.

2. Literature Review on the Use of Graphing Calculator (GC) and Geometers’ Sketchpad (GSP)

2.1 Research on the Use of Graphing Calculator (GC)

The meta-analysis studies on the usage of graphing calculator provide strong empirical evidence that hand-held technology can play an important role in mathematics instruction (Ellington, 2003; Hambree &
However, we are still continuing seeing inconsistencies in research findings in this area. Further, despite almost four decades of research showing the benefits of calculator-enhanced curricula, these hand-held technologies has not yet reached their full potential in mathematics education. In general, more research is uncovering the specific areas of mathematics that are helped by graphing calculator use and those areas that are hindered by this technology.

Further, almost all research reports are in line with the meta-analysis studies (Dunham, 2000; Burrill et al., 2002; Kastberg & Leatham, 2005). In summary, from the research literature it is evident that generally the results have favoured the use of calculating grapher in Malaysian mathematics education. Specifically, the use of graphing calculator is beneficial in improving levels of students’ mathematics achievement. Most of the studies were conducted in secondary schools and a few studies involved undergraduate students at university level. In general, more research is uncovering the specific areas of mathematics that are helped by graphing calculator use and those areas that are hindered by this technology.

In Malaysia, the Curriculum Development Centre has introduced the graphing calculator in the early 1990s but graphing calculator is still not used broadly (Muhd. Khairiltitov Zainuddin, 2003). Furthermore, there are not many schools which have explored the use of the technology (Noraini Idris, 2004). Their use in the Malaysian mathematics classroom is still very much in its infancy (Lim & Kor, 2004). According to Muhd. Khairltiv Zainuddin (2003), until the year 2003 there is no research or report of the usage of graphing calculator in schools in Malaysia. However, in response to the latest reform in the Malaysian Secondary School Integrated Mathematics Curriculum which calls for the need to integrate technology in teaching and learning of mathematics, research on graphing calculator has been gradually developed in Malaysia (Noraini Idris, 2004, 2008, Rosihan & Kor, 2003; NorAIN Mohd. Tajudin, 2011).

Generally, the reports indicate that hand-held graphing technology can be an important factor in helping students develop a better understanding of mathematical concepts, score higher on performance measures, and raise the level of their problem solving. However, most of the reports also suggest that the maximum potential for hand-held technology has not been explored. Those studies that have been reported only provide a starting point for efforts to better understand how to effectively use the technology in the classroom. According to Burill et al. (2002),

> Individual projects look at specific pieces of the picture, but the pieces do not make a coherent whole and, in fact, often seem unrelated (p. 55).

From those effectiveness studies in Malaysia and other countries, it is not really clear what causes the improvement in scores when the graphing calculator is used. Further, contradictions in opinions, inconsistencies in research findings, and endless myths phenomena are still continuously being debated in this research area. Thus, further rigorous research is needed. We must find ways to ensure that the new recommendations for hand-held technology of the NCTM Standards 2000 as well as the aims of Malaysian mathematics curriculum in introducing the technology in the classroom are achieved.

### 2.2 Research on Geometers’ Sketchpad (GSP)

This GSP software enables the construction and the animation of interactive mathematical model to be used and explored by teachers and students (Mahmud et al, 2009). The features in this software opened up space for dynamic image construction which can be manipulated, analysed, conjectured and tested. Research findings have revealed that the learning of mathematics with GSP was made easier compared to the conventional method (Teoh & Fong, 2005). Ministry of Education, Malaysia has bought the GSP license in 2004, hence, enabled the mathematics teachers in secondary schools to use the software in the teaching and learning process. The application of the GSP has given the opportunities to students as well as educators not only to enhance their skills and knowledge in using the computers but also to explore the potentials of GSP. It was spelled out in the Form Four and Form Five Additional Mathematics syllabi that the uses of computer technology, dynamic geometry software, courseware courses, internet and graphic calculator were encouraged in the teaching and learning activities.

The use of GSP software with exploratory technique was suggested in many teaching and learning of Mathematics activities to enhance the understanding of mathematical concepts (Stacey, 2007). In addition, the use of dynamic geometry software enhanced many aspects of mathematics learning. Among them was to strengthen the understanding of variables and function, to clarify the understanding of problems, to produce simulation as well as motivate the learning of Algebra. Research under the topic of Circular
Measurement discovered that the software had many advantages (Marzita & Rohaidah, 2004). One of the advantages of GSP software was its ability to allow students to explore geometric features without erasing or redrawing the figure. Automatic calculation can be done for angles, side length and ratio while adjustment of the drawing was being made. It also enabled user to build, measure and manipulate what was presented on the screen as well as giving immediate feedback when the size and shape of the object is changed (Hannafin & Scott, 1998).

GSP software was popular because of its potential in helping teachers to carry out teaching and learning by testing conjecture on geometrical shapes, relation and transformation (Kurz et al, 2005). The measurement presented on the screen will also change when users manipulate the object. Users can drag and change the position of the object without redrawing thus giving more time for users especially students to think about geometry rather than wasting time reconstructing the diagram. Consequently, this allows students to explore the possibilities of something which is difficult to perform if they were to depend on textbook, paper and pencil. Such activities evidently increase the cognitive competency. Teoh & Fong (2005) demonstrated that the teaching and learning using dynamic visualisation approach helped students to better understand the mathematical concepts taught.

3. Objectives and Research Questions

The objectives of this study are:
1. To develop the GC-GSP Modules in teaching and learning of Straight Lines topic.
2. To investigate the pedagogical usability of the GC-GSP Module in teaching and learning of Straight Lines topic.

The following are the research questions:
1. What is the appropriate modules using GC and GSP in teaching and learning of Straight Lines topic?
2. Does the GC-GSP Module met the requirements for the pedagogical usability criteria?

4. Methodology

This research is divided into two phases. The first phase is conducted to develop the appropriate modules using GC in teaching and learning of mathematics in Malaysian secondary schools for a Straight Lines topic. Instructional ADDIE Model is used in developing the GC Module. It is a five-phase instructional design model consisting of Analysis, Design, Development, Implementation, and Evaluation. For content validation, the module was submitted to a mathematics lecturer and two Form Four mathematics teachers.

The second phase employed the quantitative survey data-gathering method to determine the pedagogical usability of the modules. A sample of 30 mathematics teachers from three different schools in the district of Sabak Bernam, Hulu Selangor and Kuala Selangor are randomly selected to participate in this study. The participants (30 mathematics teacher) are introduced to the content of the module as well as the technical aspects of the GC. The participants then are given two weeks to try out the module in their schools. After two weeks, they are asked for feedbacks by responding to the pedagogical usability questionnaire. The pedagogical usability questionnaire is adapted from Nokelainen (2006). The criteria involved are student control, student activities, objective oriented, application, value added, motivation, knowledge value, flexibility and response. This instrument used a Likert scale with a range of strongly agree to strongly disagree. A total score is calculated by assigning a value of 1 (strongly disagree) to 5 (strongly agree) to each item and then adding the values. The reliability indices of the instruments were 0.88.

The data are analysed using descriptive statistics such as mean, standard deviation, percentage, and frequency. In this study, scores ranging from 3.00 to 4.00 in a 5-point scale questionnaire indicated a moderate level, whilst scores from 4.00 to 5.00 indicated high level. This is based on the general rule provided by Nugent, Sieppert and Hudson (2001) that scores can be conceived as reflecting a magnitude continuum, higher scores are indicative of greater magnitude and lower scores indicative of lower magnitude.
5. Results and Discussions

5.1 GC-GSP Modules
The ADDIE model was used to develop the GC-GSP module. In the first phase, the analysis involved the researcher identified the learning problem, the goals and objectives, the audience's needs, existing knowledge, and any other relevant characteristics to the teaching and learning processes. These analyses also considered the learning environment, any constraints, the delivery options, and the timeline for the project. For the second phase, namely the design phase, a systematic process of specifying learning objectives was conducted. The main feature of this phase was that they were guided using the balanced approach (Waits & Demana, 2000) in learning the topic. In this GC-GSP based learning environment, students were provided with tasks that required them to perform mathematical exploration, investigation and problem solving. The tasks provided students with opportunities to use mathematical tools systematically in solving problems and be engaged in investigation and exploration of patterns and relationship to help them enhance their reasoning abilities and communication skills. Finally the development phase was the actual creation (production) of the content and learning materials based on the design phase. The module was validated by two specialist teachers of mathematics teachers and a lecturer of mathematics education. Appendix A gives the details of two activities in the GC-GSP modules.

5.2 Pedagogical Usability of GC-GSP Modules
Samples to determine the pedagogical usability aspects consisted of 30 teachers; 6 (20%) are males and 24 (80%) are females. They are from different educational backgrounds in which a total of 20 (66.67%) teachers are Bachelor of Education graduates and 10 (33.33%) Postgraduate Diploma graduates. Pedagogical usability illustrated how the materials function in simplifying the learning content delivered. The usability criteria were evaluated through the eleven items constructed. The results were shown in Table 1.
Overall, the mean score for the level of pedagogical usability for the GC module in teaching and learning of Straight Lines topic is ranged between 3.93 to 4.60. The overall mean score for pedagogical usability criteria is 4.30 (SD = 0.61). This shows that the module have a high pedagogical usability criteria. This gives an indication that teachers agree that this module can be used in the classroom to teach the topic of Straight Line. Out of the 11 items, the highest mean value of 4.60 (SD = 0.50) was item 11 (this module is to add value to the teaching and learning). The mean is in the range of 3.67 to 5.00. This suggests that teachers have a high perception of the developed module as an added value for using GC in learning the Straight Line topic. The lowest mean score is item 8 (module allows students to check their performance) with mean of 3.93 (SD = 0.83). This mean value is also within the range of 2.67 to 5.00. Overall, the teachers have high opinion on GC module because it allows students to check their performance when learning the topic of Straight Lines.

The findings give indication that the module can be used in the process of learning mathematics for a Straight Lines topic. This means that the integration of GC and GSP in the teaching and learning process were appropriate. This study confirms earlier study conducted by Hennessy (2000), who demonstrated that learning graphs in Mathematics through the use of technology simplified learning and increase students understanding and confidence. Similarly, analysis from Teoh and Fong (2005) gave positive feedback on the use of Mathsoft in the learning of movements on straight lines which were also related to graphs. Their
findings showed that the graph visualisation method using technology could enhance student understanding, thus achieved better level of learning. The finding of this study was also parallel with the findings of Gray (2008) which revealed that the use of GSP Software enables the construction and animation of interactive mathematical model to be used and explored by teachers and students to improve engagement, transform the learning environment, and enhance understanding. The study conducted by Haslina et al (2000) discovered that the multimedia elements in Mathematics could interest students and increase students' performance in Mathematics. The module developed in this study has successfully integrated information technology and communication in the teaching of Straight Lines. The use of GSP software and GC in this module is suitable in plotting graphs and other dynamic simulations. The facilities are suitable and can make learning more interesting. This use of exploratory technique in the module also allowed students to use their creativity to explore new shapes, conjectures and solutions. This technique was suggested in many teaching and learning of mathematics activities to enhance the understanding of mathematical concepts (Stacey, 2007). In addition, the module developed is learner-centred in nature, hence helping teachers' awareness in increasing flexibility to accept learners' autonomy in shifting from teacher-centred to learner-centred pedagogy.

From this study, it is recommended that integrating the use of GC and GSP should be made part of a routine instructional strategy in teaching and learning of mathematics at Malaysian secondary level. However, its implementation at the classroom level needs to consider learning the technology prior to learning the subject area. This is to ensure that the characteristic of human architecture of limited working memory when dealing with novel material was not ignored. In addition, it is also recommended for teachers to make best use of this technology by employing the “balanced approach” as suggested by Waits and Demana (2000). This approach can be achieved by routinely employing three strategies that were recommended by solves analytically using traditional paper and pencil algebraic methods, and then supports the results using a GC or GSP, solves using a GC or GSP, and then confirms analytically the result using traditional paper and pencil algebraic methods, and solves using GC or GSP when appropriate (because traditional analytic paper and pencil methods are tedious and/or time consuming or there is simply no other way!). It is hoped that this approach will exploit the fullest advantages of the use of GC and GSP in helping students to achieve in-depth understanding of mathematical concepts and facilitating students in solving mathematical problems.

6. Conclusion

Overall, the GC-GSP Module developed that integrates a dynamic geometry software, GSP and the technology tool, GC can be used as materials that integrated technology in the teaching the Straight Lines topic. The module has also met the requirements of the pedagogical usability criteria, namely the student control, student activities, objective oriented, application, value added, motivation, knowledge value, flexibility and response. Teachers might no longer spend time in constructing materials using this software and tool as they can use this module. In addition, with the use of this module, students can be more investigative as constructivism approach was implemented in the teaching process in understanding the mathematical concepts, thus can help boost higher order thinking skills. Findings from this study provide evidence of pedagogical impact in incorporating the latest trends in mathematics education, namely, integrating the GC and GSP software to maximize the mathematical and pedagogical benefits to Malaysian students.

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References


**APPENDIX A**

**SUBTOPIC 5.2: Understand the concept of gradient of a straight line in Cartesian Coordinates**

**Learning outcomes:**
Students will be able to
Determine the relationship between the value of the gradient and the:
(i) steepness; and
(ii) direction of inclination of a straight line.
Materials:
1. Short notes 1
2. GSP software and Graphic Calculator TI-83 Plus
3. Worksheet 1 and Worksheet 2
4. Evaluation sheet

ACTIVITY 1
1) Work in pairs for this activity.
2) Press File menu and select New Sketch
3) Press Graph menu and select Show Grid
4) Press Graph menu and select Snap Points
5) Use the Straightedge Tool to draw a straight line from point A(0,0) to point B(4,1).

6) Press Measure menu and select Slope. The slope of straight line AB will appear as Slope AB=0.25

7) Repeat step 3 to 6 to draw other straight lines from the origin to C(3,3), D(2,5), E(-3, 4) and F(-4,2).
8) Complete the table below.

| Lines | $(x_1, y_1)$ | $(x_2, y_2)$ | Horizontal distance $|x_2 - x_1|$ | Vertical distance $|y_2 - y_1|$ | Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
|-------|--------------|--------------|-------------------------------|-------------------------------|---------------------------------|
| $AB$  | (0,0)        | (4,1)        | 4                             | 1                             | $m = \frac{1}{4}$               |
| $AC$  |              |              |                               |                               |                                 |
| $AD$  |              |              |                               |                               |                                 |
| $AF$  |              |              |                               |                               |                                 |
| $AE$  |              |              |                               |                               |                                 |

9. Rank the steepness of the straight lines in ascending order.

10. What can you conclude about the value of the gradient with the steepness of the line?

11. State the lines with positive and negative gradient. What is their direction of inclination?

12. What about the straight lines EA and FA. What can you conclude about the value of the gradient and the direction of inclination of a straight line?

**ACTIVITY 2**

**Objective:** To relate the value of the gradient with the steepness and direction of inclination of a straight line.

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<tr>
<th>Steps</th>
<th>Procedures</th>
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<th>Notes</th>
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</table>
| 1     | Clear all previous data in your GC. Make sure all format have been activated.  
• Press 2ND, [MEM] and select ‘7: Reset’  
• Press ENTER an select ‘1: All Ram’  
• Press ENTER and select ‘2: Reset’ | TI-84 Plus  
2.43  
RAM cleared |       |
| 2     | Set window settings by pressing WINDOW. Insert values such as:  
• Xmin = -6  
• Xmax = 6  
• Xscl = 1  
• Ymin = -1  
• Ymax = 6  
• Yscl = 1  
• Xres = 1  
Then, press GRAPH |  |
3. Press 2ND, [Draw], then select ‘2: Line(’, and press ENTER.

4. Use the arrow key to move the cursor to point O(0, 0), then press ENTER.

5. Use again the arrow key to move the cursor to the end point A(5, 1), then press ENTER.


7. Repeat step 3 to 6 to draw other straight lines from the origin to B(4, 4), C(2, 5), D(-3, 5) and E(-5, 2). Complete table 1.
8. Rank the steepness of the straight lines in ascending order. Complete the fourth column of table 1.

9. What can you conclude about the value of the gradient with the steepness of the line?

10. State the lines with positive and negative gradient. Complete the fifth column of table 1. What is their direction of inclination? Complete the sixth column of table 1. What can you conclude about the value of the gradient and the direction of inclination of a straight line?

<table>
<thead>
<tr>
<th>Straight Line</th>
<th>Vertical Distance</th>
<th>Horizontal Distance</th>
<th>Gradient</th>
<th>Steepness Rank</th>
<th>Positive/Negative Gradient</th>
<th>Direction of Inclination</th>
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Table 1