The Mathematics Teacher Formation and the Teaching for Research and Study Course

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ABSTRACT

The study of didactic devices with the characteristics of RSC in the teachers' mathematics formation at an Argentine university has led to the future teachers elaboration of their own ideas about of the management of the mentioned device at the Argentine high school. In this paper, results of a semi-structured interview to future teachers are shown. The teachers were attending the last year of the teaching training college at an Argentine university. Our intention was to know the receipt that the future teachers have in the management of an education under the characteristics of RSC to manage the above device in their educational practices. One of the difficulties that the future teachers signal is to select the problematic question that allows to stimulate and to provoke the whole process of study. The choice of the mentioned question is assigned to the same teachers. This constitutes a limitation as for the comprehension and later management of an education for RSC.

1. Introduction

The teacher formation is a problem of didactic research and, therefore, an open problem that different researchers have contributed with partial answers and, in many cases based and confirmed experimentally (Artaud, Cirade&Jullien, 2011; Azcárate, 2004; Bosch&Gascón, 2009; Font, 2011; Polo, González, Gómez&Restrepo, 2011; Rico, 2004; Robert&Pouyanne, 2005; Ruiz&Sierra, 2011; Sanchez&García, 2004; Sierra, Bosch&Gascón, 2012).

Godino (2009) indicates that the researches have realized various and numerous contributions in relation to the formation of mathematics teachers, but he emphasizes that it does not exist an agreement the investigation literature to designate with an unique expression the set of knowledge, competitions, dispositions, etc. that the mathematics teacher must use to favor the learning of his students. On the other hand, Bosch and Gascón (2009) indicate that the description of necessary praxeological equipment of the mathematics teacher, as well as the study of his viability and evolution conditions in the different institutions that the teacher must frequent, it is and must remain always as a main and opened problem for the research in mathematics didactics. If the teacher formation was based to provide knowledge with theory and practise components, the formation would convert in monuments that the students have to know, although it is not clear why they were built, and what they are used for. In this sense, there is a risk of wanting to form teachers leaving the responsibility of the *to apply* these equipments to the concrete situations which they are going to meet. The formation of the teachers, instead of centring on the teacher praxeological equipment, it must be situated in the heart of the professional formation the questions, difficulties or problems which the mathematics teachers have to confront in their practices, and the teachers' formation must contribute ways of approaching them.

One of the umbilical problems which the current processes of formation faces, is that the systems of education are governed by *the visit of knowledge paradigm* (Chevallard, 2001a, 2004a, 2004b, 2006) that is characterized for preferring the study of certain constructions praxeological (the *monuments*) to the study of the questions, problems or needs that are in the origin of the formation project. In this paradigm, it prevails what Chevallard names *teacher pedagogy* (Marietti, 2010), which is characterized by the

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introduction of the notions of study controlled constantly by the teacher. And the students are invited *to visit* these bodies of knowledge as a monument is visited that is not of their own. According to Chevallard (2007) it is necessary an *epistemological and didactic revolution* that puts in the beginning of the mathematics learning the study of questions with real reason to be and those that are strengthen for answering. It proposes to introduce in the education systems *functional* study processes, where the knowledge is not monuments that the teacher *shows* to the students, but material and conceptual tools useful to study and to solve problematic situations. The solution proposed to the problem takes the form of a didactic device developed by Yves Chevallard: *Research and Study Course*(RSC). Although, it is a proposal to give a solution to the current issue of the mathematics education, it is a device that is in the experimental phase, and in many occasions it has been tested in not typical education courses, such as workshops parallel to the courses(Barquero, 2009; Fonseca, Pereira&Casas, 2010; García, Bosch, Gascón&Ruiz, 2005; Rodríguez, Bosch&Gascón, 2007, 2008; Ruiz, Bosch&Gascón, 2007; Serrano, Bosch&Gascón, 2007).

To come to a pedagogy of RSC it is necessary a *school paradigm of the world questioning*, which implies basically the study of sufficiently rich, alive and fecund questions, which provoke in the students the need to continue learning, and that facilitates to open a investigation process, which allows to explore, to surmise and to validate. Chevallard (2009a, 2009b) indicates that there is not still a didactic school organization to fully develop this new paradigm: the specific infrastructures needed for an education for RSC are not available yet. This reveals that there is a need of major investigations in the area that allow strengthening the formation teacher for the design, implementation and evaluation of didactic devices with these characteristics. Therefore, though the development of didactic devices with characteristics of RSC, may not be implemented in their fullness, we do consider possible incorporating some elements that the above mentioned pedagogy involves.

In this work we present the beginning of an investigation on the didactic formation of the mathematics teacher at an Argentine national university. The presence of the notion of RSC in the mathematics teachers' formation in last two years, led the future teachers to elaborate praxeology concerning the previously mentioned didactic device. Especially, we will deal on the receipt that the education for RSC has in future mathematics teachers. In this work response is given to the following question: *which are the umbilical questions that future mathematics teachers face at the moment of interpreting an education for RSC?*

2. Theoretical framework

In this work we adopt as theoretical framework the *Anthropological Theory of the Didactic* and its last development (Chevallard, 1999, 2004b, 2006, 2007, 2009b, 2012; Ladage&Chevallard, 2010). Following the recent lines of investigation that the theory, proposed appears the need of introducing in the education systems *functional* processes study, where the knowledge do not constitute *monuments* that the teacher *show* to the students, but material and conceptual tools, useful to study and to solve problematic situations. The *Research and Study Activities* and the *Research and Study Course* are devices proposed to face the process of monumentalization of the knowledge to bring alive what Chevallard names the pedagogy of the investigation in the mathematics lessons.(Ladage&Chevallard, 2010).Next, we characterise the devices.

2.1 The Research and Study Activities

The *Research and Study Activities*(RSA) emerge as *didactic model* to approach one of the emergent issues in the current educational institutions: The *relative incompleteness of the mathematical school organizations* (Bosch&Gascón, 2010). The RSA are devices that take again the concern of the functional reconstruction of the mathematical knowledge as response to certain fundamental questions. Hereby, it is intended to exceed the binary classic structure that is characterized by the presentation of technological–theoretically elements and then tasks as *means* for the application of the first ones.

A RSA is, to begin with, a didactic organization where a class, under the leadership of a teacher *Y*, is going to make students *X* study, rebuild and make approachable a certain *Local Mathematics Organization*[3](LMO).

[3]Chevallard (1999) introduced the distinction of different types of OM, according to the degree of complexity of his components:
Punctual Mathematic Organizations (PMO): They are generated by what is considered in the institution as the only type of task and are defined from the practical - technical block.
Local Mathematic Organizations (LMO): It is the result of integrating diverse punctual praxeology. All local praxeology is characterized by a technology that serves to justify, explaining, to relate and to produce the technologies of all the punctual praxeology that integrate.

For this it is necessary to begin from a generational question *Q* which its study leads to the reconstruction of the principal elements of the initial LMO. In other terms, the RSA provokes the formation, in the bosom of a class [*X, Y*], of a didactic system S (X; Y; Q) whose purpose is the production of a response $R^{\bullet}$. Hereby, the RSA constitute a process of study praxeologically finished, since it is imposed the condition of which $R^{\bullet}$ contain the principal components of an LMO previously determined and known in advance ($R^{\diamond}$), *labeled* by the school institution. Hence, it is obtained the following formula of the RSA functioning:
$S(X; Y; Q) \rightharpoonup R^{\bullet} \approx R^{\diamond}$
This scheme comes from the *scheme herbatian* introduced by Chevallard (2008). In the process of study of *Q* it can be found other praxeologicalentities. These compose the didactic means constituted by the set of the resources mobilized by the class [*X, Y*] to study the question *Q*, to produce the response $R^{\bullet}$and to *validate* it. This praxeological complex is named means (M) and its role is indicated in the *herbartian schema semi developed*:
$[S(X; Y; Q) \rightharpoonup M] \rightharpoonup R^{\bullet} \approx R^{\diamond}$
Here the didactic system S (*X; Y; Q*) *makes* the means *M* from already existing resources of its internal and external environments or from resources created in its bosom. Working in this means, it is going to be elaborated and validated $R^{\bullet}$.

The pedagogy of the RSA raises to the teachers' formation the problem of the search of a reason to be of the curricular LMO and the need to consider the above mentioned reason to be to be a central element of the didactic organizations. Hereby, in the education system a *change will be needed in the educational profession*, since the *topos* of the professorship would incorporate a new function: the need to know some functionality or reason to be of the curricular mathematics organization. In addition, the pedagogy of the RSA needs also modifications in the *topos* of the student, since it offers the possibility of taking part in the management of almost all the didactic moments, which in the monumental pedagogy was assigned exclusively to the teacher.

2.2 The Research and Study Course

One of the fundamental issues of the RSA pedagogy is that it provokes a process of study praxeologically finished. So, although there is an implicit response before a question *Q*, it finishes for weaken the question making it appear as a means and not as an end of its own study. Furthermore, provided that the RSA are in the local level (of the *topic*), they do not constitute an effective tool to question those aspects of the school monumentalism epistemology that operates, at least, to level of the discipline and beyond(Bosc&Gascón, 2010). Especially, it does not allow to overcome the *teacher´s thematic autism* (Chevallard, 2001b). So, often the questions that constitutes the reason to be of an LMO are beyond of the local level, even beyond the *regional*, *sectorial* and up to *disciplining* level. Here, the generational question *Q* is imposed by didactic needs and proposed by the teacher (the students do not take part in the exposition and formulation of the question) depending on the LMO that is the aim of the RSA. Hereby, a new type of *didactic model arises*: *The Research and Study Course* (RSC), as need of fundamenting the didactic organizations on a really functional epistemology.

The RSC pedagogy questions elements of the school traditional contract: the teacher like the *knowledge temple*, as responsible only one of the validity of the answers, as manager of the didactic time, and the individual character of the learning. These elements remain replaced with the model of a process of collective study, directed by a teacher who shares with the group of students the responsibility of the management of the different didactic moments. The aim of the study comes defined as a set of questions *Q* which the community of study proposes to contribute a response $R^{\bullet}$. The departure point of the RSC is a generational question *Q alive* for the study community and whose response does not turn out to be directly accessible. This response must constitute itself the same significant contribution, in the sense of extending the praxeological universe of the study community. In this model, during the study activity, it will be mobilized all those resources, means, knowledge and available answers ($R^{\diamond}$) that are necessary to construct $R^{\bullet}$. Hereby, will it end up by including generally praxeology at least local, integrating praxeológical elements that can go beyond the regional and even disciplinary level?

Regional Mathematic Organizations (RMO): They are obtained by the coordination, joint and later integration of diverse local praxeology to a mathematical theory jointly.
Global Mathematic Organizations (GMO): They arise when several regional praxeologías add from the integration of different theories.

Along of the RSC, the study of the generational question Q evolves and gives place to many new *derivative questions*: Q_1, Q_2,...Q_n whose relevancy must be constantinexamineal. The essential criterion to decide the relevancy of the Q_i is its aptitude to provide praxeológical answers R_i that helps to elaborate the final response $R^{\bullet}$. It´s a constituent of RSC to take in to consideration the study of these derivative questions, that is to say, the possibility, for the study community, to provide appropriate of suitable means of validation and of arranging *medias* in the matter. For this reason, all RSC presents an opened and indeterminate structure at the beginning, since it is the process of study that is delimiting the possible ways continuing. Also, along the RSC the generational question can evolve and transform in one or several questions, for example when the advance of the study needs a new exposition or reformulation of the initial problem, which marks another degree of opening of the RSC.

3. Methodology

This paper reports results of a qualitative study and the proposed research design is the case study (Rodriguez, Gil &Garcia, 1999). Our purpose was to explore the teaching received by RSC for high school, at the last year of the teaching training college at an Argentine national university. The proposal responds to the expectation of deepening future teachers' ideas about teaching by RSC, to make suggestions on teachers training, with the prospect of making their future classroom daily work an experience of encounter and dialogue with their students, and, making of the research a permanent activity of searching, learning and self-training.

The study was conducted with two students, who are characterized for having a good academic performance throughout their career. Each future teacher is identified as FP_1 and FP_2. We highlight that FP_2 has never developed teaching activities in high school, while FP_1, when developing our research has been doing teacher activities at an Argentine high school for 6 months.

The math teacher curriculum that FT_1 and FT_2 has a duration of four years, which aims to influence in the training of future teachers as follows:

- The basic concepts for the formation of a teacher in mathematics, as well as information on various fields of expertise and some issues of applied mathematics,
- The current epistemological conception and historical development of them in relation to mathematical knowledge,
- Pedagogical, psychological and didactic concepts necessary to transpose this knowledge into the classroom at all levels of the education system.

In the last year of mathematics teaching curriculum, it is intended to make teaching practices whose purpose is: to allow the design and implementation of didactic sequences and solving specific problems that arise in the classroom and the deepening of an attitude of ongoing reflection on the practice itself. We note that throughout the future teacher formation, this instance is the first encounter that the future teacher has with high school students, taking their topos as a teacher.

Following the guidelines proposed in the mathematics teacher curriculum, we did our research when future teachers should make their teaching internships. We note that during training, they studied the Anthropological Theory of the Didactics, its latest developments and research related(Barquero, 2009; Bosch, Espinoza & Gascón, 2003; Bosch, Fonseca &Gascón, 2004; Bosch, García, Gascón & Ruíz, 2006; Chevallard, 1999, 2007; Chevallard, Bosch & Gascón, 1997; Cid&Bolea, 2007; Llanos, Otero & Bilbao, 2011; Ruíz, Bosch & Gascón, 2007). Although future teachers were involved in the study of new developments in ATD, the praxeological didactic model in their training, both mathematical and pedagogical, is characterized as traditional. The university institution in which the future teachers belong is characterized by being strongly marked by the paradigm of knowledge monumentalization (Corica, 2010). Pretending that future teachers develop substantiated practices in the pedagogy of investigation or questioning of the world, involves profound changes in their perspectives on the mathematics teaching, being that while they studied the ATD, all their education (including university) is based on the paradigm of knowledge monumentalization. Adhere to pedagogy research involves placing the questions in the central axis of the study process, allowing students to work on an ongoing inquiry. This promotes collective forms of social construction of knowledge, which now not only the scientific community is accredited to participate. The complex issues, needs and living conditions nowadays require an education that places the research as an important part of its work, linking it to the classroom as a means of learning to increase the students'

interest on learning, and research, creative and observation, skills to deepen understanding of these realities, and probably, later, may also contribute to its improvement. One possibility for this purpose is to think and develop an education based on the wealth of the pedagogy of questioning the world.

In this paper we report results of a semi-structured interview (Rodríguez, Gil&García, 1999) carried on with both future teachers, in order to know their thoughts on the design and implementation of instructional devices with features of RSC and RSA at the Argentine high school. Whereas our aim is to circumscribe RSC teaching, we consider relevant to deepen on the difference between RSA devices and RSC because though, they are gestated to face the problems of knowledge monumentalization, between the two devices there is an essential fracture in the study means (M) management, which makes of the RSC a device that would allow a genuine functional study of mathematics in high school.

The interview lasted 70 minutes and was audio recorded. It was structured in four episodes, demarcated by generational questions around the teaching devices RSC and RSA, and its possible implementation in the Argentine high school. The generational question that we pose at first place is:

Q_0: What are the essential features required at an Argentine high school to implement a teaching by RSC and RSA?
This question led to another four fundamental questions that characterize these educational devices, and that demarcated each one of the episodes that structured the interview:
Q_1: What are the essential characteristics of learning devices RSA and RSC?
In the first episode of the interview, we sought to know what future teachers understand on the structural characteristics of an RSA and RSC. While the RSA are didactic devices whose structure limits and conditions the student activity compared to what happens to a teaching by RSC, the RSA allow functional reconstruction of mathematical knowledge and install some elements of the pedagogy of world questioning. Furthermore, we believe that the comparison between the two devices highlights the distinction between an essential component, which is the genesis and management of the means study.
Q_2: How to manage these devices at the argentine high school?
This issue was dealed in the second episode of the interview, and sought the views of prospective teachers under what conditions could didactic devices be developed with features of RSA and RSC at Argentine high school.
$Q_{2,1}$: How to design and manage the means of study for RSA and RSC?
$Q_{2,2}$: What chronogenetic and topogenetic modifications are generated in the process of studying the introduction of educational devices with the features of a RSC and RSA?
These last two questions were addressed in episodes 3 and 4 of the interview. We consider that the questions are subordinated to Q_2 and they search highlighting changes at amesogenesis, chronogenesis and topogenesis level, which involve the implementation of the didactics devices with characteristics of RSC and RSA. Especially, here the accent is placed on the mesogenetic change that implies the genesis and management of the means of a device or another one. So, one RSC particularly is an original and new device in the sense that corrupts with the idea of delimiting and limiting great part of the genesis and management of the means in the hands of the teacher, transferring to the students. This necessarily carries necessarily to changes to a chronogenesis and topogenesis level.

4. Results and analysis

Next, we present the results obtained of the analysis of the interview. Though one of the future teachers had a brief educational experience at school, we do not find substantial differences as for his interventions in the interview. Rather, discussions of ideas were provoked to arrive between them both of at a consensus.

For the presentation of results, in first place we indicate the initial question of the interviewer (I) that gave origin to the discussions between the future teachers, and then the corresponding analysis. Every shift of speech, both of the interviewer and of the interviewed one it was labeled by a number, which is indicated in each of the fragments of protocols proposed later, to the effects of realizing the analysis of the interview.

4.1 Characterization of the RSA and the RSC

In this paragraph we will limit to indicate the response obtained of the future teachers to *Q_1: What are the essential characteristics of learning devices RSA and RSC?* During the episode 1 it was requested to the

future teachers who were indicating the essential characteristics of the RSA and the RSC. This question generated that the future teachers specify which the reasons are for those who proposed the didactic devices. Since they indicated:

[4] I: *What are the essential characteristics of learning devices RSA and RSC?*

[5]FT_1: (…)*In the school the problem arises from what is studied. Answers are given directly to the questions.*

[6]FT_2: *There the problem of the knowledge monumentalization (…) Why are we studying this topic. Only it gives you the answer and not questions maybe current that we make them to take these answers and to need these answers.*

[8]FT_1: *One sees as a monument, as a finished work. As if you are going to visit a monument.*

In this instance it was requested to FT_2 to extend his response, with the intention of knowing what he refers to with the following expression: *Only it gives you the response and you do not ask may be current that we them do that they take them to these answers and to needing these answers*. The future teacher indicated:

[11]FT_2: *(…)What happens is that I was recounting … along the history like the things are happening and changing (…) That they are the same that generated this knowledge.*

[13]FT_2: *May be there are other questions that also have to do with this to knowledge nowadays that they are more current.*

[15]FT_2: *That gives them sense… Reason to be still.*

In the first instance, the future teachers could identify which is the reason to be of the RSA and the RSC. That is to say, they identified which is the issues which the mathematics education faces and why the mentioned didactic devices were developed.

On the other hand, we infer that for FT_2 a mathematical question can be studied with sense in the school if mathematical legitimacy has, that is to say, if it appears in certain umbilical situations of the mathematics. This needs of major deepening, so in order that the study of a question makes sense in the school asks from itself that not only it has mathematical legitimacy, but also *social or cultural* and *functional legitimacy* (Gascón, 2003). In turn, the study of *current problems* that FT_2 mentions, not necessarily they imply that they are more relevant and interesting to the most ancient problems that gave origin to the mathematical knowledge. Here we identify one of the issues ones who face the educational formation, which is to know the functionality or reason to be of the praxeology proposed in the curriculum design. May be this idea is deepened when we extend our investigation, which has as an intention that the future teachers elaborate didactic devices with the characteristics the RSC and face the issues of managing the offers in courses of the high school.

Since in the discussion supported up to the moment, the future teachers did not characterize to the didactic devices, the interviewer continued inquiring over the difference between the same ones:

[18]I: *Then, Why are the RSA and the RSC proposed?*

[19]FT_1: *They are born because of the problem that in the school the question is not studied. There is no question.*

[21]FT_1:*It is studied directly. For the knowledge monumentalization the RSC born. Not, the RSA. They arise from a generational question.*

[24]FT_2: *What happens is that the RSA is like more bounded that the RSC.*

[25]FT_1: *Because the RSA is the activity that is done. A question is arise(…) First it was the RSA and the RSC later… It changed activity to tour.*

[26]I: *Is there a difference between these didactic devices?*

[28]FT_2: *(…)The RSC is wider actually (…) the RSA answers to a type of question only.*

[30]FT_2: *And the RSC I feel that it separates it has other things, it is as the RSA is very stuck to the praxeology, as it says, well, I have a question, a response to this question by means of a technique, a technology and a theory, and there a RSA always finishes. On the other hand, in the RSC it is not. It seems that we study other things (…) we see the different course that a student can have. The different questions can arise other questions that from these questions, that can include other topics (…) tour of study and research is like it tends to involve more all the topics.*

According to the future teachers, the RSC would have an overwhelming status in relation to the RSA for its functional legitimacy. The RSC would allow to face the thematic autism phenomenon (Chevallard, 2001b), since it would facilitate to study in a *connected* way the questions that are studied at the school, whereas the RSA would have a character of punctual, that would not allow the latter study. Likewise, the future teachers could detect one of the issues that involves the pedagogy of the RSA: to produce the encounter of the study

community with a finished praxeology, reducing its action to what the teacher limits. Nonetheless, here it is not clear if for the future teachers the process of study, which involves an education by RSC, implies moving along pre-established answers, and the construction of the final response requires the transition from different RSA. This characteristic of RSC responds to the limitations of the RSA, since it integrates them at the time that it completes its *epistemological functions*. One of the characteristics of the RSA is that it can integrate different punctual praxeology in a local praxeology, with the limitation of not being able to motivate the transition from a local praxeology to another one, nor to articulate them among themselves. In one RSC, the production of the final response always requires being able to relate the questions derived from Q, which *entails* the construction of increasingly wider and more complete praxeologies.

4.2 Management of RSA and RSC devices at the Argentine high school

In this second paragraph we show the analysis of the answers obtained from the future teachers to Q_2*: How to manage these devices at the argentine high school?* Next, we indicate the interventions of the future teachers to the mentioned question:

[31]I: *What differences do you find in managing a RSA and a RSC at the Argentine high school? Could they be applied to the school?*
[32]FT_1:*Not The RSA. Because it does not answer to a question, actually.*
[33]I: *No?*
[34]FT_2: *Yes, it does.*
[39]FT_1: *May be The difference between the RSC and the activity … it might be that the RSA is a question and a whole… a whole surrounding punctual and local organization that answers this. However, the tour is more. I have a generational question and it goes away since questions are put in different branches and it goes away. Obviously you should not lose …*
[40]FT_2: *To what response you want…*
[41]FT_1: *To what response I want to come. But maybe the RSC is more… everything is formed and everything is dismantled. That there are no… there are no…*
[42]FT_2: *Actually.*
[43]FT_1: *That there are no gaps.*
[45]FT_1: *Right, it is like, the RSA is more punctual.*
[46]FT_2*: The RSC is more designed. The RSA, well the RSA also but …*

Here the future teachers continued resolving the differences between both didactic devices, since in the previous episode this aspect had remained incomplete. Nonetheless, we detect that confusion arises as regards the a priori design of every device. On the one hand, the RSC would be provided with a more superior potential than the RSA, but, on the other hand, RSC seems that the totally is designed a priori, and that from a beginning would have limited a final praxeology which the study community should reconstruct. We attribute this condition of the future teachers to all their formation. Thus they have always been exposed to an education limited by the paradigm of the monumentalization knowledge. Therefore, this resistance to the interpretation of the radical changes in the education that implies an education for RSC seems logical. Since the future teachers did not give response to the initial question of this second episode the interviewer continued inquiring:

[47]I: *What do we do in the school?*
[48]FT_1: *I feel that the RSA is more punctual. That in the school it is necessary to go more with a tour because it is more dynamic.*
[49]I: What you mean with dynamic?
[50]FT_1: *That it can change. Or, you can improvise more. I mean, you are at the school, a question arises, well. You base on this question. You develop the question. Without caring about the rest what want but it is as that you crumble everything what you are going to teach, the whole content. The whole block of questions. I feel that the RSA is a part of the RSC.*
[55]I: *Do you understand the RSA as little parts of a RSC, and the RSC as a wider device, more* Comprehensive?
[56]FT_1: *Always with a question.*
[57]I: *It Always arises from a question.*
[58]FT_2: Sure*, the RSA. They all depart from questions, but clearly, the RSA refers to something in particular*
[59]FT_1: *Sure and that the RSC was done like more saying, well not as an activity. A tour throughout questions of the whole content allows the RSC.*

From this second episode we distinguish that the future teachers coincide that both devices arise from questions, and that the difference is that in the RSA the point of departure is the organization that is intended to come and, from there it is elaborated the question whose study allows such a reconstruction. This discussion allowed extending incomplete aspects of the characteristics of the didactic devices, which we were indicating in the first episode of the interview.

Although in this second episode the future teachers continued penetrating into the differences between both devices, they be still the characteristics of the means of study that involves every device were not still clear, as well as their management in the classroom.

4.3 Management of the study means of the RSA and the RSC

In the third episode of the interview it was intended that the future teachers give response to $Q_{2,1}$*: How to design and manage the means of study for RSA and RSC?* Next, it is presented fragments of the interview that give response to the mentioned question:

[60]*I: Given the characteristics of the RSA and RSC, What would you propose to the students for each of them?*
[61]FT_1*: And I believe that a problem.*
[62]*I: A problem.*
[63]FT_1*: Right (…) a problem that they have to face. Although they feel puzzled, you tell them, okay we leave it. It might be also, let's put it in this little part …They can analyze it, and, I don't know, throughout two weeks, for example, it is already with other tools…*
[64]FT_2*: It is as if the first one is a very strong question.*
[65]FT_1*: That is why it cannot be answered.*
[66]*I: The problem or the original question that gives origin to this problem. What do we propose to the students?*
[70]FT_1*: A question, but a very wide one.*
[72]FT_1*: According to Brousseau, I would be saying to them what I want, if I take the question to him.*
[73]*I: Brousseau?*
[74]FT_1*: Right, because I am always thinking about a problem. Not saying what I want. I feel that I go and say, "well, boys the question is this, now we are going to answer it". No, I feel that I am saying everything I want. I have to take them a problem arises this question … A problem that arises the question.*
[79]FT_2*: I feel that we cannot say to them yet. We are going to have to solve first small things. like it is going to be breaking down and then, suddenly, I have the response to the initial problem.*
[80]*I: And what difference do we have with the Didactics Situations Theory proposed by Brousseau?*
[81]FT_2*: I feel that with Brousseau I have the isolated problem.*
[82]FT_1*: I feel that a RSA is with Brousseau. It is like it goes a problem and the answer. One problem and the answer.*
[83]FT_2*: As if in the RSC a question looks first for generational of everything.*
[84]FT_1*: As a regional praxeology*
[85]I*: And there?*
[87]FT_2*: And from there we are looking for small things, for things that are going out to questions. Asif we are weaving to go to the response. On the other hand I feel that in Brousseau not. In Brousseau we raise a problem. Always we have an aim to where we want to go with this problem, ready we do what we have to do … we get there…*
[89]FT_2: *(…) That is why in the RSC there are different ways. There is no only one, it is not that we are going to have to go thereabouts. It is as in Brousseau's situation it is as that already we have determined the way, even though it could change a bit. Not with the response of the pupils but with what we are going to say or not. It is as if the tour is already made. And in the RSC it is freer*
[144]I: *Then, let's imagine taking these ideas to the school. Which would be more suitable? Why?*
[145]FT_1: *I don't know if the RSA Is okey. It is like you say well, what do I want to teach? I see the answer and there I make the perfect answer.*
[149]FT_2: *Delimited.*
[150]FT_1: *Well isomorphic or point by point. And the RSC here clearly says …*
[152]FT_2: *I propose a question and the answer will be seen where it gets.*
[153]FT_1: *A tour. A question opened to come to this answer. Where are we going to cross to come to this answer? Where* in which direction are we going to go?
[154]FT_1: *Like it does not sound as a heading.*

The future teachers continued penetrating on the characteristics of the RSA and RSC in relation to the didactic device developed by Guy Brousseau (1986). We emphasize that these future teachers have been formed in first place in the framework of the Didactic Situations Theory proposed by Guy Brousseau (1986). Due to this fact emerges their need to compare the didactic devices and to establish differences. Here the future teachers perceive ~~to~~ the didactic situations as devices that gestate a didactic system S (*X; Y;?; M*) amputated, since they do not find explicit the questions *Q* that give origin to the means proposed by the teacher. On the other hand, although the future teachers demonstrate the existence of certain similar characteristics between the Didactic Situations Theory and the RSA, they consider relevant that in the latter device it should be formulated the question that gives origin to the means. On the other hand, in the RSC what prevails is the question *Q* that will give place to certain way of study.

In this episode the future teachers penetrated into the essential characteristics of the didactic devices, but it did not emerge the characteristics of the immersed student topos in an education based on these devices. This was deepened in the following episode.

4.4 Topogenetic and chronogenetic modifications

The fourth episode focused on discussing aspects related to the students and teacher topos, inside the process of study with the characteristics of didactic devices like RSC and RSA, with the intention of giving response to $Q_{2,2}$*: What chronogenetic and topogenetic modifications are generated in the process of studying the introduction of educational devices with the features of a RSC and RSA?* Next, we indicate~~d~~ the manifestations of the future teachers:

[164]I: *Which is the student topos in the considered didactic devices? Are there differences?*
[165]FT_1: *Be responsible for the questions. Find technique.*
[188]FT_2: *What happens in the RSA is that we propose a question and the students are going to start searching and well. We are going to have type of tasks regarding this question. And well, the students are going to start looking for techniques to solve this type of tasks. It would be like before. I see it similar to what it was going on before. [It refers to the Didactic Situation Theory proposed by Guy Brousseau (1986)] (…) The tour lets, allows that the students raise questions (…)*

To this response, the interviewer continued asking on the way of working of students involved in an education by RSA or RSC. According to the future teachers, for both devices it is required that the students appropriate of the resolution of the problems, so that the paradigm of questioning the world establishes, while the knowledge visits paradigm goes away(Chevallard, 2012). Having this new paradigm in which the students would be, we questioned the future teachers on how they would manage the questions proposed by the students.

[175]I: *How would students raise the questions? What would the teacher do?*
[176]FT_1: *Yes, it is necessary to guide them a little because sometimes they ask many things and if I allow them to ask everything they want you never finish.*
[179]I: *We have a set of questions proposed by the students, what do we do?*
[180]FT_2: *We have to solve them, but we are going to see first what the questions are. We are not going to answer all the questions because some of them may be interesting to the initial question, but others may not. What do we do? Do we try to solve everything and then we realize that those were not useful?*
[181]I: *And how do we answer? Who answers them?*
[182]FT_2: *Everyone*
[183]I: *And with what*?
[192]FT_2: *Right, the generational question. I thought that they were going to ask me a question. No, they are several. They can be several.*
[194]FT_2: *Right, but in the example that I was giving. You see that several questions are made, the generational question of a point. He says that if we know the size of the population in some periods of time, can we predict what will happen after n periods of time? … What hypothesis on the population and its environment is assumed? How can forecasts be done and how can they be validated? I imagined the problem as a question.*
[196]FT_2: *No. anyway, they have to see. I already realized. We cannot give this to the students.*
[197]I: *Why?*
[198]FT_2: *Because I feel that they are going to feel drowned, puzzled… with so many questions, without having a top of of something to start resolving.*

[202]FT_1: *I would put them. It would not be. I mean, it would be a wide and then a less wide question. They would get lost for me.*
[204]I: *No, and what is more, to give them something where the students have to ask. Are they used to ask? I do not refer to ask why, but to make relevant questions for the problem proposed. To think about other possibilities of what they are giving us.*
[205]FT_2: *They have to get involved. It has to interest…them*
[206]FT_1: *It is necessary to make the modality of work explicit; to say that what we are implementing has a different modality and that we are departing from these questions.*
[232]I: *What does the student do? How do we answer to these questions?*
[233]FT_1: *I, personally, at the end of the class, I would ask them to write down the questions and I would take them home.*
[234]I: *But for example, two hours writing down questions?*
[235]FT_1: *No, because for me there has to be a purpose. I don't know if my class would be a hundred percent RST. I mean, because you put a generational question, how do we answer it? Let's suppose that they have a response, not to come to the conclusion that is not so easy … and to ask themselves, just for today, where they want to start. Where they start making questions, we use them. That they remain with some questions. When certain questions arise we say, well, this is what we are going to answer may be, that would seem…*
[236]I: *And how are we going to answer it?*
[237]FT_1: *They investigating/searching. They may bring books.*
[240]I: *But it is necessary to take the responses of all of them. Altogether we are making these answers…*
[241]FT_2: *(…)Here it is like more uncertain.*
[243]FT_2: *For the teacher, because here, what are they going come up with? And often you to have to think if answering the question that they are making to you is going to be fruitful or is going to make you lose time…*
[245]FT_2: *It is difficult. In the meanwhile you have many factors… that they may get lost…*
[246]FT_2: *They have to be completely…*
[247]FT_1: *Completely engaged.*
[248]FT_2: *Yes, very engaged. For that reason, we have to give them a problem in which they are really interested. That they want to answer, not that you feel interested in.*

From the manifestations of the future teachers, we highlight that the need of a change in the didactic traditional contract in high school was installed, which would be required to make explicit to the students. In one RSC the traditional contract of the problem types*?* Is broken, which is characterized by the relatively frequent change from a type of problems to another one and, therefore, certain inflexibility in the use of the mathematical techniques. What is sought is a deepened study of a field of problems. In this new pedagogy, the means M would consist of the questions and answers proposed by the students. But also the interventions of the future teachers let permeate certain difficulties at the moment of managing an education by RST. Since they manifest the need that the teacher should be the one who selects the questions that the students propos. It emerges the need of being the teacher who *controls* the action of the students and knows the way, which generates that RST is reduced to the study of a finished praxeology and subordinated to a pedagogy that is not far from the RSA. We attribute these ideas of the future teachers to all their formation at the high school and university, which is governed by the paradigm of the monumentalization of the knowledge and for a teacher characterized for controlling the students work.

Here are again installed the difficulties that the future teachers are going to face when managing devices with the characteristics of RSC. The activity of the teacher not only involves the suitable selection of questions Q to bring to the classroom, but also to manage its study. Installing the new pedagogy of the world questioning requires deep changes in the topos of the student, who should also be prepared to fight with long periods of uncertainty. Mesogenetic changes imply necessarily topogenetics and chronogenetics changes.

5. Final reflections

One of the fundamental issues, and to an institutional level, that the future teachers face when contemplating an education by RSA and by RSC is the necessary change of the didactic contract prevailing in the schools. This change is linked directly by the students and teacher topos: they must assume new tasks,

and, among them, negotiate the new responsibilities that they will have to take. For the future teachers, the change in relation to the teacher role, involves fundamentally to choose the problematic question that allows to stimulate and to provoke the whole process of study of the project that it generates, and must be kept alive along the same process?. It should be a powerful question that could be presented as a challenge for the student, who must accept it.

In the pedagogy of RSC, it is necessary that the community of study focuses, during a long period of time, on the study of the same question, which keeps it *alive* and *opened* in the subsequent classes, and, that in addition, could derive from the study new questions. Furthermore, the relevancy of these questions and the opportunity (or not) of considering its study must appear as one more gesture of the process of study, to be negotiated between the teacher and the students. The latter is an indispensable aspect, which is reduced by the future teachers interviewed, since they demonstrate the need of being the teacher the one who selects, of all the proposed questions, those to be studied. It is necessary to penetrate into the formation of the teachers in this respect, since it is a fundamental factor that goes against the pedagogy of the RSC.

The future teachers proved to be enthusiastic and committed with the development of an education by RSC, and as it was indicated, they highlight the need of the students to take *responsibility* for the study of the questions. We emphasize that this does not imply the need of contributing with answers as a mere pretext to show the usefulness of the new taught knowledge, but it transports to the need of *inspecting vast regions*, and this to leads to the possibility of finding *unexpected entities* and, in this way, finding *small elements* that allows to make progress in the study. This faces directly the concern of the teachers for knowing always in advance the tour of the process of study of their students.

The adhesion to this limitation is made explicit in the interview with the future teachers, that, in spite of having being involved in the study of the RSC pedagogy in their last years of the teaching training college, they denote in their manifestations the attachment to the paradigm of the knowledge monumentalization. The pedagogy of the RSC modifies the dominant school tradition pedagogy, with the foundation of *protecting* the students from *dispersion* and *chaos*, by providing means immediately adaptable to the programs of study, controlled and fed by the teacher. It is necessary to invite to the study community to defend the subsequent answers provided, even though they still have a provisional character and they are still attached to a process of active study.

These fundamental issues, linked to the activity of the mathematics teacher, will be deepened in our near investigations. Since it is suggested that the future teachers design a device with characteristics of RSC and implement it at the Argentine high school. This activity will allow to penetrate into the professional problematic that emerge from the reported interview, and they will allow to permeate different ones, such as guiding a RSC. Proposing an education by RSC needs to move along seven dialectics (Chevallard, 2007, Marietti, 2010) in courses used to a mathematics education marked by the knowledge monumentalization paradigm. It proposes to study how the future teachers can lead these *gestures of study and research* that were introduced as a praxeological equipment during his educational formation, with the intention of answering to the restrictions that the dominant pedagogy imposes on the school life of the mathematical activity.

In this line, after the implementation that the future teachers realize, it is proposed to ponder on the difficulties detected in the implementation of its devices, altogether with their training teachers and to modify its offers for future implementations. We consider this stage of reflection to be vital in the teachers' formation, since from the professional training it is necessary to approach in a real and effective way the questions that the exercise of the teaching considers in mathematics, and not only consisting of a preparatory course of the mathematical and pedagogic available knowledge at the moment. There is needed of a work of narrow cooperation between the school system that it constitutes the *area* of the educational activity, the didactic investigation that acts as source of question and production of praxeological resources for the renewal and improvement of this activity and the *profession of mathematics teacher* itself, on which lays in last instance, the duty of identifying the needs, which are in constant evolution, and which its members must face.

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